

General Analysis of U -Spin Breaking in B Decays

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Abstract

We analyse the breaking of U -spin on a group theoretical basis. Due to the simple behaviour of the weak effective hamiltonian under U -spin and the unique structure of the breaking terms such a group theoretical analysis leads to a manageable number of parameters. Several applications are discussed, including the decays $B \rightarrow J/\psi K$ and $B \rightarrow DK$.

1 Introduction

Non-leptonic decays of bottom hadrons play an important role in the investigation of CP violation. While this effect has been established in non-leptonic Kaon decays, and the CKM mechanism of CP violation is consistent with what is seen in non-leptonic B meson decays, the observations cannot be easily linked with the fundamental parameters.

The reason for this is the lack of a reliable method to compute the amplitudes of non-leptonic decays, which are given by matrix elements of an effective interaction expressed in terms of a combination of four-quark operators between meson states. The QCD dynamics turn out to be so complicated, that currently neither factorization based methods nor lattice calculations yield reliable and precise predictions.

While non-leptonic decays are a nice laboratory for studying QCD methods, the road to precise predictions for CP violation in non-leptonic B decays seems to be the use of flavour symmetries, supplemented by the enormous amount of data expected from LHC and the (Super) flavour factories. From the current perspective this will remain true for some time, until a qualitative breakthrough is achieved in the field of QCD methods.

There is a vast literature in the field of flavour symmetries and their applications to B decays. The complete decomposition of the two-body B decay amplitudes in terms of irreducible flavour $SU(3)$ matrix elements has been performed in [1, 2]. Applications to B decays have been considered in [3–8], and we shall use the notation of [9]. Furthermore, flavour-symmetry strategies related to the extraction of CP violating parameters have been discussed in [9–12].

The main problem with flavour symmetries is that they hold only approximately. The usual starting point is flavour $SU(3)$, which suffers from a substantial breaking due to the sizable mass of the strange quark [13]. On the other hand, the isospin subgroup is known to have a much smaller breaking, and can be safely assumed to be unbroken in this context. Hence one may consider the other two possibilities to identify $SU(2)$ subgroups of flavour $SU(3)$, which run under the names U -spin and V -spin. Among these two subgroups, the generators of U -spin, under which the d and the s quark form a fundamental doublet, commute with the charge operator, which makes this subgroup particularly interesting with respect to electroweak interactions. On the other hand, U -spin symmetry is broken at the same level as the full flavour $SU(3)$ due to the splitting $m_s - m_d$.

However, this breaking has a simple structure and can be readily included by a spurion analysis. In the present paper we perform such an analysis and apply it to non-leptonic B decays. We make use of present data to extract the symmetry-breaking matrix elements and give relations which include U -spin breaking, which can be tested in the future, once more data are available.

2 Flavour Symmetries

U -spin is an $SU(2)$ subgroup of the full $SU(3)$ flavour symmetry group, in which the d and the s quark form a doublet. A priori, U -spin is as badly broken as the full $SU(3)$, since the masses of the

two quarks are substantially different:

$$\Delta m \equiv m_s - m_d \sim \Lambda_{\text{QCD}} ,$$

where Λ_{QCD} denotes the nonperturbative QCD scale. Regarding the group structure of the breaking term, we observe that the relevant mass term in the Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= m_d \bar{d}d + m_s \bar{s}s = \frac{1}{2}(m_s + m_d)(\bar{d}d + \bar{s}s) + \frac{1}{2}\Delta m(\bar{s}s - \bar{d}d) \\ &= \frac{1}{2}(m_s + m_d)\bar{q}q - \frac{1}{2}\Delta m\bar{q}\tau_3 q , \end{aligned} \quad (2.1)$$

where

$$q = \begin{pmatrix} d \\ s \end{pmatrix}$$

is the U -spin quark doublet. Thus we conclude that the breaking term can be described as a triplet spurion

$$\mathcal{H}_{\text{break}} = \frac{1}{2}\Delta m\tau_3 = \epsilon B_0^{(1)} , \quad (2.2)$$

where $B_0^{(1)}$ is an irreducible tensor-operator with $j = 1$ and $j_3 = 0$ of U -spin. Here we also introduce the small quantity ϵ related to the symmetry breaking $\epsilon \sim \Delta m/\Lambda_{\chi SB}$ where $\Lambda_{\chi SB}$ is the chiral symmetry breaking scale.

If we consider a matrix element of some operator $\mathcal{O}(x)$, which can be decomposed into irreducible tensor-operators of U -spin, we may consider U -spin breaking to leading order by evaluating

$$\langle \tilde{f} | \mathcal{O}(0) | \tilde{i} \rangle = \langle f | \mathcal{O}(0) | i \rangle + (-i) \int d^4x \langle f | T[\mathcal{O}(0)\mathcal{H}_{\text{break}}(x)] | i \rangle + \dots , \quad (2.3)$$

where the states \tilde{f} and \tilde{i} include the breaking term, while the states f and i are the U -spin symmetric states. A general analysis of U -spin breaking can be performed by a group theory analysis of the breaking term by decomposing the T product of the operator \mathcal{O} with $\mathcal{H}_{\text{break}}$ into irreducible tensor operators $T_{j_3}^{(j)}$ of U -spin.

The simplest non-trivial case emerges if the operator \mathcal{O} is an U -spin doublet, which we denote by $\mathcal{O}_{j_3}^{(1/2)}$. In this case, the last term in (2.3) decomposes into

$$\begin{aligned} (-i) \int d^4x T[\mathcal{O}_{\pm 1/2}^{(1/2)}(0)\mathcal{H}_{\text{break}}(x)] &= (-i\epsilon) \int d^4x T[\mathcal{O}_{\pm 1/2}^{1/2}(0)B_0^{(1)}(x)] \\ &= \sqrt{\frac{2}{3}} \left[K_{\pm 1/2}^{(3/2)} \mp \sqrt{\frac{1}{3}} K_{\pm 1/2}^{(1/2)} \right] . \end{aligned} \quad (2.4)$$

Aside from the trivial example of the currents $j = \bar{u}\Gamma q$, $q = d, s$, also the effective weak hamiltonian for B decays is a pure U -spin doublet, even if electroweak penguins are included. The latter is true due to the fact that the s and the d quark carry the same electroweak quantum numbers. Thus

from the group theoretical point of view we may decompose the weak effective hamiltonian into its irreducible tensor components according to

$$H_{\text{eff}}^{\Delta C=\pm 1} = \frac{4G_F}{\sqrt{2}} \left[V_{cb}V_{ud}^* P_{1/2}^{(1/2)} + V_{cb}V_{us}^* P_{-1/2}^{(1/2)} \right], \quad (2.5)$$

$$H_{\text{eff}}^{\Delta C=0} = \frac{4G_F}{\sqrt{2}} \left[V_{cb}V_{cd}^* Q_{1/2}^{(1/2)} + V_{ub}V_{ud}^* R_{1/2}^{(1/2)} + V_{cb}V_{cs}^* Q_{-1/2}^{(1/2)} + V_{ub}V_{us}^* R_{-1/2}^{(1/2)} \right], \quad (2.6)$$

where the operators $P_{j_3}^{(1/2)}$, $Q_{j_3}^{(1/2)}$, and $R_{j_3}^{(1/2)}$ are renormalization group invariant combinations of four-quark operators.

In the following we shall use this group theoretical decomposition to discuss U -spin and its breaking in various B decays. To this end, we have to identify the U -spin multiplets of hadronic states. Starting from the definition of the fundamental quark doublets (we use the same sign convention as in [9]),

$$\begin{bmatrix} |d\rangle \\ |s\rangle \end{bmatrix} = \begin{bmatrix} |\frac{1}{2} + \frac{1}{2}\rangle \\ |\frac{1}{2} - \frac{1}{2}\rangle \end{bmatrix}, \quad \begin{bmatrix} |\bar{s}\rangle \\ |\bar{d}\rangle \end{bmatrix} = \begin{bmatrix} |\frac{1}{2} + \frac{1}{2}\rangle \\ -|\frac{1}{2} - \frac{1}{2}\rangle \end{bmatrix}, \quad (2.7)$$

we obtain for the decaying B mesons

$$|B^+\rangle = |(u\bar{b})\rangle = |0, 0\rangle, \quad \begin{bmatrix} |B^0\rangle = |(d\bar{b})\rangle \\ |B_s\rangle = |(s\bar{b})\rangle \end{bmatrix} = \begin{bmatrix} |\frac{1}{2}, +\frac{1}{2}\rangle \\ |\frac{1}{2}, -\frac{1}{2}\rangle \end{bmatrix}. \quad (2.8)$$

The mesons in the final state are in terms of U -spin

$$\begin{bmatrix} |K^+\rangle = |(u\bar{s})\rangle \\ |\pi^+\rangle = |(u\bar{d})\rangle \end{bmatrix} = \begin{bmatrix} |\frac{1}{2}, +\frac{1}{2}\rangle \\ -|\frac{1}{2}, -\frac{1}{2}\rangle \end{bmatrix}, \quad (2.9)$$

$$\begin{bmatrix} |\pi^-\rangle = -|(\bar{u}d)\rangle \\ |K^-\rangle = -|(\bar{u}s)\rangle \end{bmatrix} = \begin{bmatrix} -|\frac{1}{2}, +\frac{1}{2}\rangle \\ -|\frac{1}{2}, -\frac{1}{2}\rangle \end{bmatrix}, \quad (2.10)$$

$$\begin{bmatrix} |K^0\rangle = |(\bar{s}d)\rangle \\ \sqrt{3}/2 |\eta_8\rangle - 1/2 |\pi^0\rangle = |(\bar{s}s - \bar{d}d)\rangle \\ |\bar{K}^0\rangle = |(\bar{d}s)\rangle \end{bmatrix} = \begin{bmatrix} |1, +1\rangle \\ |1, 0\rangle \\ -|1, -1\rangle \end{bmatrix}, \quad (2.11)$$

$$\begin{bmatrix} |K^{*0}\rangle = |(\bar{s}d)\rangle \\ 1/\sqrt{2} |\phi\rangle - 1/2 |\rho^0\rangle - 1/2 |\omega\rangle = |(\bar{s}s - \bar{d}d)\rangle \\ |\bar{K}^{*0}\rangle = |(\bar{d}s)\rangle \end{bmatrix} = \begin{bmatrix} |1, +1\rangle \\ |1, 0\rangle \\ -|1, -1\rangle \end{bmatrix}. \quad (2.12)$$

From this we derive the decomposition of the neutral states

$$\begin{aligned} |\pi^0\rangle &= -\frac{1}{2} |1, 0\rangle + \frac{\sqrt{3}}{2} |0, 0\rangle_8, \\ |\eta\rangle &= \sqrt{\frac{2}{3}} |1, 0\rangle + \frac{\sqrt{2}}{3} |0, 0\rangle_8 - \frac{1}{3} |0, 0\rangle_1, \\ |\eta'\rangle &= \frac{1}{2\sqrt{3}} |1, 0\rangle + \frac{1}{6} |0, 0\rangle_8 + \frac{2\sqrt{2}}{3} |0, 0\rangle_1, \end{aligned}$$

$$\begin{aligned}
|\rho^0\rangle &= -\frac{1}{2}|1,0\rangle + \frac{\sqrt{3}}{2}|0,0\rangle_8, \\
|\omega\rangle &= -\frac{1}{2}|1,0\rangle - \frac{\sqrt{3}}{6}|0,0\rangle_8 + \sqrt{\frac{2}{3}}|0,0\rangle_1, \\
|\phi\rangle &= \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{6}}|0,0\rangle_8 + \frac{1}{\sqrt{3}}|0,0\rangle_1,
\end{aligned} \tag{2.13}$$

where the subscript 1, 8 on the two U -spin singlet states refers to the $SU(3)$ transformation properties of the corresponding state.

It is interesting to note that one may infer some relations in the U -spin limit. These have been discussed in the literature [5, 14], but are rederived here in a different way. The key observation is that due to CKM unitarity all CP violation in the standard model is proportional to the Jarlskog invariant

$$\text{Im}\Delta = \text{Im}(V_{cb}V_{cd}^*V_{ub}^*V_{ud}) = -\text{Im}(V_{cb}V_{cs}^*V_{ub}^*V_{us}). \tag{2.14}$$

In particular, all CP violating rate differences $\Delta\Gamma = \Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})$ are proportional to $\text{Im}\Delta$. Exchanging the roles of the d and the s quark will flip the sign of $\text{Im}\Delta$ in $\Delta\Gamma$ as can be seen in (2.14).

The relation (2.14) may be combined with the group theory of U -spin. We note that we have for the operators in the effective hamiltonian

$$[U_{\pm}, Q_{\mp 1/2}^{(1/2)}] = Q_{\pm 1/2}^{(1/2)}, \quad [U_{\pm}, P_{\mp 1/2}^{(1/2)}] = P_{\pm 1/2}^{(1/2)}, \tag{2.15}$$

where U_{\pm} is the operator which raises or lowers the 3-component of the U -spin by one unit, respectively.

For the case of charged B mesons we have a U -spin singlet in the initial state, and hence the final states may only have $U = 1/2$. Using (2.15) we see that

$$\langle B^+ | Q_{-1/2}^{(1/2)} | f, 1/2 \rangle = \langle B^+ | Q_{+1/2}^{(1/2)} | f, -1/2 \rangle, \tag{2.16}$$

and the analogous relation for the matrix elements of $P_M^{(1/2)}$. In the effective hamiltonian these matrix elements appear with CKM factors in which the role of the s and d quarks are interchanged and hence we get in the U -spin limit

$$\Delta\Gamma(B^+ \rightarrow (f, U_3 = 1/2)) = -\Delta\Gamma(B^+ \rightarrow (f, U_3 = -1/2)) \tag{2.17}$$

for any state f .

The neutral B mesons form a U -spin doublet, and hence the possible final states can be either a singlet or a triplet. Using the above reasoning, we infer the relation

$$\Delta\Gamma(B_d \rightarrow (f, U = 0, U_3 = 0)) = -\Delta\Gamma(B_s \rightarrow (f, U = 0, U_3 = 0)). \tag{2.18}$$

Finally, the case of a triplet final state yields also similar relations for the $U_3 = \pm 1$ components. Using again (2.15) we get

$$\Delta\Gamma(B_d \rightarrow (f, U = 1, U_3 = 1)) = -\Delta\Gamma(B_s \rightarrow (f, U = 1, U_3 = -1)). \tag{2.19}$$

These relations may serve as a test for the amount of U -spin breaking. In fact, rewriting the relation for the rate differences into the usual observables we get

$$\frac{A_{\text{CP}}(\#1)}{A_{\text{CP}}(\#2)} = -\frac{\Gamma(\#2)}{\Gamma(\#1)}, \quad (2.20)$$

which we shall check once we will discuss applications. However, one has to keep in mind, that these relations also reduce the number of independent observables.

Finally we remark that we can supplement the U -spin relations by isospin symmetry. As pointed out above, we consider it safe to use isospin as an exact symmetry in this context and use this to constrain the U -spin breaking parameters. The fundamental doublets are defined as

$$\begin{bmatrix} |u\rangle \\ |d\rangle \end{bmatrix} = \begin{bmatrix} |\frac{1}{2} + \frac{1}{2}\rangle \\ |\frac{1}{2} - \frac{1}{2}\rangle \end{bmatrix}_I, \quad \begin{bmatrix} |\bar{d}\rangle \\ |\bar{u}\rangle \end{bmatrix} = \begin{bmatrix} |\frac{1}{2} + \frac{1}{2}\rangle \\ -|\frac{1}{2} - \frac{1}{2}\rangle \end{bmatrix}_I, \quad (2.21)$$

which fixes our conventions.

However, under isospin the effective hamiltonian decomposes in a more complicated way. The transformation properties of the operators defined in (2.5) are given in table 1. Note that for the $Q_{\pm 1/2}^{(1/2)}$ operators the only topology contributing with $\Delta I = 1, 3/2$ respectively is the electroweak penguin.

Operator	$(\Delta I, \Delta I_z)$
$\Delta C = 1 :$	
$P_{+1/2}^{(1/2)}$	$(1, -1)$
$P_{-1/2}^{(1/2)}$	$(1/2, -1/2)$
$\Delta C = 0, b \rightarrow d$	
$Q_{+1/2}^{(1/2)}$	$(1/2, -1/2) \oplus (3/2, -1/2)$
$R_{+1/2}^{(1/2)}$	$(1/2, -1/2) \oplus (3/2, -1/2)$
$\Delta C = 0, b \rightarrow s$	
$Q_{-1/2}^{(1/2)}$	$(0, 0) \oplus (1, 0)$
$R_{-1/2}^{(1/2)}$	$(0, 0) \oplus (1, 0)$

Table 1: Classification of irreducible U -spin operators in terms of isospin.

3 Applications

In this section we apply the above formalism to non-leptonic two-body B decays. Clearly we shall not discuss all possible decays here, rather we focus on two sample applications to check how far we can get without any restrictive ad-hoc assumptions.

As stated above, the charged B mesons are U -spin singlets and hence - due to the simple U -spin structure of the effective hamiltonian - the final state has to be a doublet or - including U -spin breaking

- a quadruplet. Considering two-body decays, this corresponds to having one final-state meson in a U -spin doublet, while the other has to be either U -spin singlet or triplet.

In the case where one of the final state mesons is in a triplet there is another complication. Since U -spin breaking is not that small, the mass eigenstates are quite different from the U -spin eigenstates, i.e. there is not even an approximate mass eigenstate corresponding to an $s\bar{s} - d\bar{d}$ U -spin state. As already described above (see (2.9) ff.), there are three mass eigenstates contributing to $U_z = 0$, which all have to be taken into account.

The two neutral B mesons form a doublet under U -spin. In the U -spin limit the contributing final states have to form either a singlet or a triplet, while we can have also admixtures of $U = 2$ once we include U -spin breaking. When considering two-body decays there are in total three possibilities. The decays into two charged final states necessarily have either $U = 0$ or $U = 1$, since the charged mesons form U -spin doublets. The neutral mesons form either U -spin singlets or triplets, in which case the two-body final states can have $U = 0, 1$ and 2 . Clearly a final state with $U = 2$ can be reached only through U -spin breaking.

In the following we discuss the general possibilities to constrain U -spin breaking. Evidently the data on B_s decays is needed which will be available in the near future from LHC data. However, based on the current data one may already discuss U -spin breaking in some modes, e.g. $B \rightarrow J/\psi (K \text{ or } \pi)$ (charged and neutral) and in the decays $B^\pm \rightarrow D (K^\pm \text{ or } \pi^\pm)$.

As will become apparent in these example analyses, the precision of the data does not yet suffice to draw strong conclusions. For some modes, this may change with LHCb, however, the modes including neutral light mesons in the final states will only be accessible with a sufficient precision at a super flavour factory. In general, the parameters appearing in our expressions can be constrained meaningfully in the relevant range with the expected precision from future experiments. We have checked this by performing some sample fits within a simple “future scenario”. The details, however, depend strongly on the mode, and in some cases one still has to resolve discrete ambiguities.

3.1 U -Spin Breaking in $B \rightarrow M_0 M_0$ and in $B \rightarrow \text{CP-Eigenstates}$

When considering decays of neutral B mesons into two neutral mesons, one has to deal with admixtures of U -spin multiplets. Using the decomposition

$$\begin{aligned} \mathcal{H}_{eff}^{b \rightarrow d} |\bar{B}_d\rangle &= -\frac{1}{\sqrt{2}} |1, 0\rangle_{d,0} - \frac{1}{\sqrt{3}} |1, 0\rangle_{d,\epsilon(3/2)} + \frac{1}{\sqrt{6}} |1, 0\rangle_{d,\epsilon(1/2)} \\ &\quad - \frac{1}{\sqrt{2}} |0, 0\rangle_{d,0} + \frac{1}{\sqrt{6}} |0, 0\rangle_{d,\epsilon} - \frac{1}{\sqrt{3}} |2, 0\rangle_{d,\epsilon} , \end{aligned} \quad (3.1)$$

$$\begin{aligned} \mathcal{H}_{eff}^{b \rightarrow s} |\bar{B}_s\rangle &= +\frac{1}{\sqrt{2}} |1, 0\rangle_{s,0} - \frac{1}{\sqrt{3}} |1, 0\rangle_{s,\epsilon(3/2)} + \frac{1}{\sqrt{6}} |1, 0\rangle_{s,\epsilon(1/2)} \\ &\quad - \frac{1}{\sqrt{2}} |0, 0\rangle_{s,0} - \frac{1}{\sqrt{6}} |0, 0\rangle_{s,\epsilon} + \frac{1}{\sqrt{3}} |2, 0\rangle_{s,\epsilon} , \end{aligned} \quad (3.2)$$

$$\begin{aligned}\mathcal{H}_{eff}^{b \rightarrow d} |\bar{B}_s\rangle &= +|1, +1\rangle_{d,0} - \frac{1}{\sqrt{6}}|1, +1\rangle_{d,\epsilon(3/2)} - \frac{1}{\sqrt{3}}|1, +1\rangle_{d,\epsilon(1/2)} \\ &\quad + \frac{1}{\sqrt{2}}|2, +1\rangle_{d,\epsilon},\end{aligned}\tag{3.3}$$

$$\begin{aligned}\mathcal{H}_{eff}^{b \rightarrow s} |\bar{B}_d\rangle &= -|1, -1\rangle_{s,0} - \frac{1}{\sqrt{6}}|1, -1\rangle_{s,\epsilon(3/2)} - \frac{1}{\sqrt{3}}|1, -1\rangle_{s,\epsilon(1/2)} \\ &\quad - \frac{1}{\sqrt{2}}|2, -1\rangle_{s,\epsilon},\end{aligned}\tag{3.4}$$

one may express all the amplitudes in terms of U -spin amplitudes. Doing this in full generality leads to a large number of independent U -spin amplitudes for $U_z = 0$ final states already in the symmetry limit, and does in general not allow for a determination of all breaking amplitudes. One theoretical exception is given by decays $B^0 \rightarrow P^0 P^0$: when both final state particles belong to the same multiplet, Bose symmetry forbids antisymmetric final states, leading to a reduction of possible amplitudes¹. However, this possibility remains a theoretical one, because in order to perform this fit, all decays of this class would have to be measured time-dependently, which seems not possible in the near future.

Choosing the subset of decays formed by $b \rightarrow s$ transitions of B_d -mesons, combined with $b \rightarrow d$ transitions of B_s -mesons [9] results in 19 parameters facing up to 18 observables, therefore in this case one additional assumption is needed.

In any case, the current situation concerning the data is insufficient to perform such an analysis, since the B_s system has not been fully explored yet. Clearly with the advent of LHC this situation will change once LHCb measures the decay rates and the CP asymmetries of the corresponding B_s transitions.

Decays into CP eigenstates (or, more generally, states which are not flavour-specific) play an exceptional role, because of the additional information coming from time-dependent measurements. Each of these decays forms a subset with its U -spin partner formed by exchanging all down and strange quarks in the process, because they have effectively only one amplitude. These subsets can be discussed separately from the rest of the corresponding class, which allows for fits with a small number of parameters, even when other decays of that class have not been measured yet. This feature has been extensively exploited in the U -spin limit, or including factorizable U -spin breaking only (see e.g. [5, 14]). In that case, the two decays in question have five independent observables (because of relation (2.20)), but only three parameters, so a fit for up to two weak phases is possible. However, these determinations suffer from the systematic uncertainty related to U -spin breaking.

Including the breaking corrections to first order for these subsets, one observes that the breaking amplitudes form only one effective breaking amplitude as well. However, again this does not suffice for an analysis of the breaking which is completely free from additional assumptions: the number of parameters increases by four, while only one additional independent observable becomes available. In these cases, for example the following two strategies may be used:

- If one amplitude is clearly dominating ($|A_1/A_2| \sim \delta$), one may consider the U -spin breaking

¹This fact has been overlooked in [9], the corrections to the corresponding decompositions are straight forward.

for the leading amplitude only, neglecting only terms of order ($\mathcal{O}(\epsilon^2)$, $\mathcal{O}(\epsilon\delta)$). This is for example the case in $B \rightarrow D_{d,s}^+ D_{d,s}^-$ decays, which are dominated by their colour allowed tree contribution.

- If one of the two parts in the leading amplitude is dominated by a colour allowed tree contribution, one may use the factorization assumption for that part only, as opposed to using it for the whole amplitude, and fit for the breaking amplitude in the other part.

In both cases, the number of free parameters increases only by two, so in principle a fit becomes possible; in addition, as one additional observable is available, one may determine that way one of the weak phases with correspondingly smaller systematic uncertainty. If for one class of decays the whole set is measured, these strategies may be used with the whole set, so the decays into flavour-specific modes can be included.

Finally, let us comment on the phenomenologically important decays $B_{d,s} \rightarrow (\pi/K)^+(\pi/K)^-$, of which a subsection are CP eigenstates. A priori, both strategies are not applicable in this case. However, there exists a small amplitude combination, corresponding to the fact that the amplitudes for $B_d \rightarrow K^+ K^-$ and $B_s \rightarrow \pi^+ \pi^-$ are “dynamically suppressed”, i.e. they proceed only via annihilation. This fact might be used by setting to zero the corrections to this amplitude combination with $\Delta I = 1/2$, thereby reducing the number of parameters to 15, while there are in principle 16 measurements available from these decays. However, a fit still requires the time-dependent measurements of all these decays. At the moment, there is no sign of large U -spin breaking in these decays [15].

3.2 The decays $B \rightarrow J/\psi (K \text{ or } \pi)$

The decays $B \rightarrow J/\psi (K \text{ or } \pi)$ are under the simplest cases of $\Delta C = 0$ from the group theoretical point of view, because of J/ψ and B^- being U -spin singlets. Our analysis is based on the data shown in tables 2 and 3. As a first step, we check for U -spin violation by testing the U -spin relation (2.20).

Decay	BR/ 10^{-4}	A_{CP}
$B^- \rightarrow J/\psi K^-$	10.07 ± 0.35	$0.017 \pm 0.016(*)$
$B^- \rightarrow J/\psi \pi^-$	$0.49 \pm 0.06(*)$	0.09 ± 0.08

Table 2: Measurements for the decays $B^- \rightarrow J/\psi(K \text{ or } \pi)$, data taken from the PDG [16]. (*): Error enhanced by the PDG because of inconsistent measurements.

Inserting the data from table 2 and neglecting a tiny phase space difference, we get

$$(A_{\text{CP}} \times \text{BR})_{B^- \rightarrow J/\psi K^-} + (A_{\text{CP}} \times \text{BR})_{B^- \rightarrow J/\psi \pi^-} = 0.22 \pm 0.17, \quad (3.5)$$

adding errors simply in quadrature. This result is not significant and a real test may only be performed, if at least one of the asymmetries is measured significantly different from zero.

In many applications naive factorization has been applied, which allows to include at least the factorizable part of U -spin breaking. In this picture one expects the ratio of branching ratios to be given only in terms of CKM factors and the ratio of form factors. One gets the theoretical prediction

$$\frac{\text{BR}(B^- \rightarrow J/\psi K^-)}{\text{BR}(B^- \rightarrow J/\psi \pi^-)} \sim \left(\frac{F^{B \rightarrow K}(M_{J/\psi}^2)}{F^{B \rightarrow \pi}(M_{J/\psi}^2)} \right)^2 \left| \frac{V_{cb}^* V_{cs}}{V_{cb}^* V_{cd}} \right|^2 = 33.9 \pm 6.1, \quad (3.6)$$

where the form factor ratio is taken from QCD sum rule calculations [17] and scaled to $q^2 = m_{J/\psi}^2$ with aid of a simple BK ansatz [18]. This has to be contrasted with the experimental number

$$\frac{\text{BR}(B^- \rightarrow J/\psi K^-)}{\text{BR}(B^- \rightarrow J/\psi \pi^-)} = \begin{cases} 19.2 \pm 1.5 & \text{(measurement of the ratio)}, \\ 21.4 \pm 1.9 & \text{(combined single measurements)}. \end{cases} \quad (3.7)$$

The sizable discrepancy indicates the well known fact that these decays have large non-factorizable contributions.

On the other hand, the data in table 2 are not sufficient to allow a fit to the general group theoretical expressions. Hence some additional assumptions are necessary. We shall assume the following:

- The amplitude proportional to $\lambda_{ud/s} = V_{ub}V_{ud/s}^*$ is expected to be small compared to the one proportional to $\lambda_{cd/s} = V_{cb}V_{cd/s}^*$, because its tree operator contribution has only penguin matrix elements. Hence we will not take into account U -spin breaking for this amplitude.
- We shall also make use of isospin symmetry. This means that we have to take into account also the decays of the neutral B modes, since they are the isospin partners of the charged B mesons. When making use of isospin, the matrix elements identified in the U -spin analysis are splitted into their two isospin components as shown in table 1. Here we neglect the contribution with $\Delta I = 1, 3/2$ proportional to $\lambda_{cs/d}$, which receive contributions from penguin matrix elements of electroweak penguin operators only; hence we assume the corresponding penguin contributions to be a pure $\Delta I = 0, 1/2$ amplitude for both the $b \rightarrow s$ and $b \rightarrow d$ transition.

Decay	BR/ 10^{-4}	A_{CP}	S_{CP}
$\bar{B}^0 \rightarrow J/\psi \bar{K}^0$	8.71 ± 0.32	$-0.002 \pm 0.020(*)$	0.657 ± 0.025
$\bar{B}^0 \rightarrow J/\psi \pi^0$	0.205 ± 0.024	0.10 ± 0.13	$-0.93 \pm 0.29(**)$

Table 3: Measurements for the decays $\bar{B} \rightarrow J/\psi(K \text{ or } \pi)$. Time-dependent measurements are taken from the HFAG [19], other data from the PDG [16]. (*): Error enhanced by the PDG due to inconsistent measurements. (**): Error enhanced according to the PDG prescription for the same reason.

For the neutral B mesons we include the data shown in table 3. Using the above assumptions, we are

lead to the following parametrization:

$$\begin{aligned}
\langle B^- | \mathcal{H}_{eff} | J/\psi K^- \rangle &= N_{J/\psi K} (1 + x_\epsilon + \epsilon e^{-i\gamma} r_0 e^{i\phi_0}) , \\
\langle B^- | \mathcal{H}_{eff} | J/\psi \pi^- \rangle &= \frac{\lambda}{1 - \lambda^2/2} N_{J/\psi K} (-1 + x_\epsilon + e^{-i\gamma} r_0 e^{i\phi_0}) , \\
\langle \bar{B}^0 | \mathcal{H}_{eff} | J/\psi \bar{K}^0 \rangle &= N_{J/\psi K} \left[1 + x_\epsilon + \epsilon e^{-i\gamma} (r_0 e^{i\phi_0} - 2r_1^K e^{i\phi_1^K}) \right] , \\
\langle \bar{B}^0 | \mathcal{H}_{eff} | J/\psi \pi^0 \rangle &= \frac{\lambda}{1 - \lambda^2/2} N_{J/\psi K} \left[-1 + x_\epsilon + e^{-i\gamma} (r_0 e^{i\phi_0} - 2r_{3/2}^\pi e^{i\phi_{3/2}^\pi}) \right] , \quad (3.8)
\end{aligned}$$

where the normalization factor $N_{J/\psi K}$ is chosen such that $N_{J/\psi K}^2 = \text{BR}(B^- \rightarrow J/\psi K^-)$ in absence of U -spin breaking and penguin effects, which implies $\epsilon = |V_{ub}V_{us}^*|/|V_{cb}V_{cs}^*|$. As a consequence, the corresponding ratios of lifetimes and phase space factors have to be taken into account when computing the branching ratios from (3.8) for the other decays. Furthermore, the ratios r_0 , r_1^K and $r_{3/2}^\pi$ are the penguin and u quark tree contributions (normalized to $N_{J/\psi K}$) respectively, which contain a factor $R_u = |V_{ub}V_{ud}^*/V_{cb}V_{cd}^*|$. Finally, the complex parameter x_ϵ represents the U -spin breaking part in the leading contribution, again normalized to $N_{J/\psi K}$. As inputs from the CKM fit we use, in addition to the ones described above, those from table 4. The results of our fit are given in table 5, the

Parameter	Global fit value
λ	0.2252 ± 0.0008
γ	$(66.8_{-3.8}^{+5.4})^\circ$
$\beta_{w/o J/\psi}$	$0.48_{-0.04}^{+0.02}$

Table 4: CKM parameters taken from [20], results as of summer 08. The lower uncertainties of γ and $\beta_{w/o J/\psi}$, which refers to the fit to β excluding the measurement of $\sin(2\beta)$ from $B \rightarrow J/\psi K_S$, have been slightly enhanced to reflect the non-gaussian behaviour of the distribution in a conservative way.

results for the U -spin breaking parameter are additionally shown in fig. 1. The fit shows three distinct solutions, two of which have $\phi_0 \sim 0$, while the third one has $\phi_0 \sim \pi$. As the solutions interfere in the fit and make it unstable, we perform two separate fits with the restrictions $\phi_0 \in [-\pi/2, \pi/2]$ and $\phi_0 \in [\pi/2, 3\pi/2]$, covering the whole parameter space.

The sizable difference between the branching ratios of the charged and the neutral $B \rightarrow J/\psi K$ modes is somewhat surprising. The isospin analysis shows that it is driven by the $\Delta I = 1$ contribution of the effective hamiltonian, which is doubly CKM suppressed. Hence the ratio between these branching ratios should be given by the ratio of lifetimes which is close to unity, modified only by a doubly Cabibbo suppressed tree contribution and electroweak penguins. In our fit, this results in $r_1^K \sim 1 (\geq 0.78@1\sigma)$, which is quite large, but on the other hand not conclusive at the moment. Furthermore, the non-vanishing central values for the CP asymmetries imply a non-vanishing value for r_0 , $r_0 \geq 0.09@1\sigma$, in combination with a non-trivial phase. However, as is obvious from the significance of the data, the allowed range at two standard deviations includes zero. Concerning U -spin

$\phi_0 \in [-\pi/2, \pi/2] (\chi^2 = 0.51)$			
Parameter	best fit value	1σ range	2σ range
$\text{Re}(x_\epsilon)$	0.08	[0.02, 0.41]	[-0.03, 0.63]
$\text{Im}(x_\epsilon)$	-0.14	$[-0.28, -0.04] \vee \leq -0.6$	unconstrained
$N_{J/\psi K}^2$	8.39	[4.60, 9.38]	[3.65, 10.17]
r_0	0.88	[0.07, 0.26] \vee [0.56, 1.47]	[0.00, 1.72]
ϕ_0	0.09	[-0.22, 0.61]	unconstrained
r_1^K	1.60	[1.18, 2.37]	[0.66, 2.85]
ϕ_1^K	-0.07	$[-0.75, -0.50] \vee [-0.17, 0.04]$	[-0.90, 0.88]
$r_{3/2}^\pi$	0.49	[0.00, 0.09] \vee [0.24, 1.18]	[0.00, 1.46]
$\phi_{3/2}^\pi$	(0.16)	unconstrained	unconstrained

$\phi_0 \in [\pi/2, 3\pi/2] (\chi^2 = 0.01)$			
Parameter	best fit value	1σ range	2σ range
$\text{Re}(x_\epsilon)$	0.13	[0.06, 0.45]	[0.01, 0.66]
$\text{Im}(x_\epsilon)$	(0.59)	≥ 0.06	unconstrained
$N_{J/\psi K}^2$	6.27	[3.76, 8.60]	[2.96, 9.93]
r_0	0.29	[0.09, 1.03]	[0.00, 1.38]
ϕ_0	2.78	[2.28, 3.25]	unconstrained
r_1^K	1.40	[0.78, 2.14]	[0.31, 2.58]
ϕ_1^K	0.57	[0.05, 0.91]	[-0.87, 1.11]
$r_{3/2}^\pi$	0.06	[0.00, 0.18]	[0.00, 0.31]
$\phi_{3/2}^\pi$	(2.55)	unconstrained	unconstrained

Table 5: Results for the fit to $J/\psi(K \text{ or } \pi)$ data, as explained in the text. The values in brackets indicate that due to a broad allowed range the central value is not significant.

breaking, the fit prefers a non-vanishing imaginary part of the U -spin breaking parameter x_ϵ , while it is not bounded from above. The first observation is due to (3.5) showing a deviation from zero, and especially preferring equal signs for the CP asymmetries, while the branching ratios, as seen above, are compatible with no breaking at all. This is again a hint to non-factorizable U -spin breaking. The reason for the second observation lies in the fact, that no observable depends in leading order on $\text{Im}(x_\epsilon)$ when assuming a power-counting $x_\epsilon, r_i \sim \lambda$. The other parameters lie within relatively large ranges, within or including the expected order of magnitude.

3.2.1 $B^\pm \rightarrow D (K^\pm \text{ or } \pi^\pm)$ decays

As an example for $\Delta C = \pm 1$ transitions we consider the decays $B^\pm \rightarrow D (K^\pm \text{ or } \pi^\pm)$. As mentioned above, these transitions are governed by a single CKM factor, since there are four different quark flavours in the final state. In particular, this leads to vanishing direct CP asymmetries in the corre-

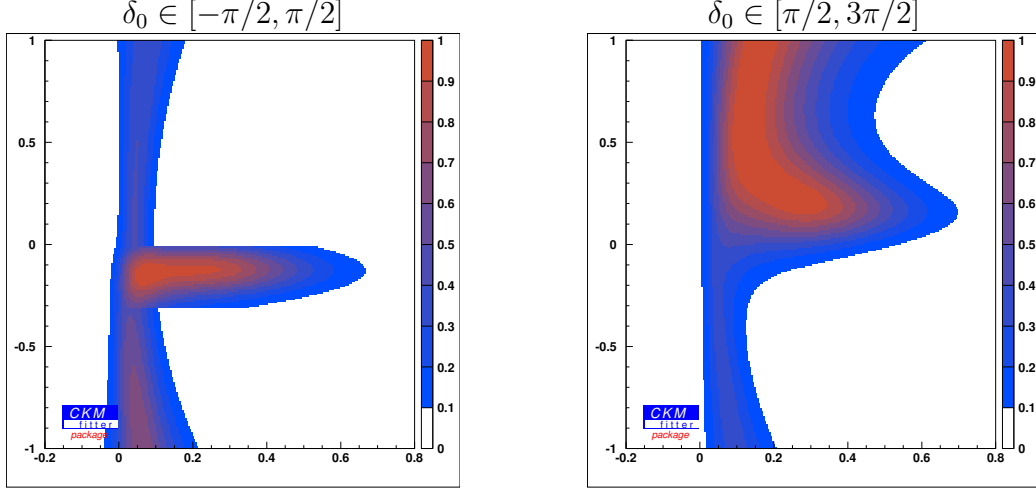


Figure 1: The fit results for the U -spin breaking parameter x_ϵ in $B \rightarrow J/\psi K$ and $B \rightarrow J/\psi \pi$ in the complex plane.

sponding decays, so the number of parameters as well as the one of observables is less by a factor of two.

However, it has been proposed a few years ago [21,22] to discuss observables from decays, where the (neutral) D meson in the final state is reconstructed in a decay mode which is a CP eigenstate. This leads to interference between $B \rightarrow D$ - and $B \rightarrow \bar{D}$ -modes, where $B^- \rightarrow DK^-$ is the “golden mode” to extract γ with negligible theoretical error. The analysis can be transferred to $B \rightarrow D\pi$ one to one, however, in this case the second amplitude, $B^- \rightarrow \bar{D}^0\pi^-$ is not only colour-, but in addition doubly Cabibbo-suppressed, which leads to very small interference effects.

Turning to U -spin, the analysis is analogous to the $B \rightarrow J/\psi(K \text{ or } \pi)$ modes, however, with only a single CKM factor in each amplitude. The parametrization in this case reads

$$\begin{aligned}
 \langle B^- | \mathcal{H}_{eff} | D^0 K^- \rangle &= \lambda \tilde{A}_1 (1 + y_{1,\epsilon} e^{i\theta_1}) , \\
 \langle B^- | \mathcal{H}_{eff} | D^0 \pi^- \rangle &= (1 - \lambda^2/2) \tilde{A}_1 (1 - y_{1,\epsilon} e^{i\theta_1}) , \\
 \langle B^- | \mathcal{H}_{eff} | \bar{D}^0 K^- \rangle &= \lambda R_u e^{-i\gamma} \tilde{A}_2 e^{i\theta_A} (1 + y_{2,\epsilon} e^{i\theta_2}) , \\
 \langle B^- | \mathcal{H}_{eff} | \bar{D}^0 \pi^- \rangle &= -\lambda^2 R_u e^{-i\gamma} \tilde{A}_2 e^{i\theta_A} (1 - y_{2,\epsilon} e^{i\theta_2}) ,
 \end{aligned} \tag{3.9}$$

a common factor $A\lambda^2$ is absorbed into the definition of $\tilde{A}_{1,2}$. In the fit, we include the (to order 1%) common phase-space factor Φ and the lifetime of the B -meson by the definition

$$A_{1,2} = \sqrt{\Phi(m_B, m_\pi) \tau_{B^-}} \tilde{A}_{1,2} . \tag{3.10}$$

Note that we choose A_1 to be real, while for A_2 one has to keep a phase because of the interference effects described below. Furthermore, the U -spin breaking quantities $y_{1,\epsilon}$ and $y_{2,\epsilon}$ are real and positive, since their phases are taken into account explicitly.

Defining now the CP eigenstates²

$$|D_{\pm}^0\rangle = \frac{1}{\sqrt{2}} (|D_0\rangle \pm |\bar{D}_0\rangle) , \quad (3.11)$$

one has the additional observables

$$\bar{\Gamma}(B^- \rightarrow D_{\pm}^0 K^- / \pi^-) = \frac{1}{2} (\Gamma(B^- \rightarrow D_{\pm}^0 K^- / \pi^-) + \Gamma(B^+ \rightarrow D_{\pm}^0 K^+ / \pi^+)) , \quad (3.12)$$

$$A_{\text{CP}}(B^- \rightarrow D_{\pm}^0 K^- / \pi^-) = \frac{\Gamma(B^- \rightarrow D_{\pm}^0 K^- / \pi^-) - \Gamma(B^+ \rightarrow D_{\pm}^0 K^+ / \pi^+)}{\Gamma(B^- \rightarrow D_{\pm}^0 K^- / \pi^-) + \Gamma(B^+ \rightarrow D_{\pm}^0 K^+ / \pi^+)} , \quad (3.13)$$

with the four relations

$$\begin{aligned} \bar{\Gamma}(B^- \rightarrow D_+^0 K^- / \pi^-) \mathcal{A}_{\text{CP}}(B^- \rightarrow D_+^0 K^- / \pi^-) = \\ -\bar{\Gamma}(B^- \rightarrow D_-^0 K^- / \pi^-) \mathcal{A}_{\text{CP}}(B^- \rightarrow D_-^0 K^- / \pi^-) , \end{aligned} \quad (3.14)$$

$$\begin{aligned} \bar{\Gamma}(B^- \rightarrow D_+^0 K^- / \pi^-) + \bar{\Gamma}(B^- \rightarrow D_-^0 K^- / \pi^-) = \\ \bar{\Gamma}(B^- \rightarrow D^0 K^- / \pi^-) + \bar{\Gamma}(B^- \rightarrow \bar{D}^0 K^- / \pi^-) , \end{aligned} \quad (3.15)$$

where the first relations require in particular opposite signs for the CP-asymmetries. Furthermore, in the U -spin limit, relation (2.20) implies

$$A_{\text{CP}}(B^- \rightarrow D_{\pm}^0 K^-) \text{BR}(B^- \rightarrow D_{\pm}^0 K^-) = -A_{\text{CP}}(B^- \rightarrow D_{\pm}^0 \pi^-) \text{BR}(B^- \rightarrow D_{\pm}^0 \pi^-) . \quad (3.16)$$

Including U -spin breaking, this leaves eight independent observables in total.

The eight observables face 7 parameters appearing in (3.9), if the weak angle γ is treated as an input, otherwise we have to deal with 8 parameters. However, one has to take into account parametric invariances: one observes one discrete invariance³,

$$\gamma \rightarrow \pi - \gamma , \quad \theta_A \rightarrow \pi - \theta_A , \quad \theta_{1,2} \rightarrow -\theta_{1,2} , \quad (3.17)$$

which leaves all observables invariant because this transformation effectively replaces every phase by its negative value. In the future, as long as γ does not lie near 90° (which is not the case, according to present data), this ambiguity is trivially resolved by the observation of other γ -dependent processes.

In addition there is one continuous invariance: One has the freedom to redefine the parametrization (3.9) in such a way, that

$$A_{1,2}(1 + y_{1,2} e^{i\theta_{1,2}}) \rightarrow A'_{1,2}(1 + y'_{1,2,\epsilon} e^{i\theta'_{1,2}}) = e^{i\theta_\xi^1} A_{1,2}(1 + y_{1,2} e^{i\theta_{1,2}}) , \quad \text{and} \quad (3.18)$$

$$A_{1,2}(1 - y_{1,2} e^{i\theta_{1,2}}) \rightarrow A'_{1,2}(1 - y'_{1,2,\epsilon} e^{i\theta'_{1,2}}) = e^{i\theta_\xi^2} A_{1,2}(1 - y_{1,2} e^{i\theta_{1,2}}) , \quad (3.19)$$

which is always possible in a restricted range for θ_ξ^i . The restriction is given by the possible values of the corresponding parameter combinations, when considering $y_{1,2} \in [0, 0.6]$ in the fit.

²In the following we neglect any mixing in the D system.

³ γ has been restricted to lie in $[0, \pi]$, which excludes additional solutions.

The experimental results for these decays are given in table 6. The two colour suppressed decays have not been measured so far, the two CP asymmetries $B \rightarrow D^0 K^- / \pi^-$ do not enter the fit, because they are zero by construction, but are given mainly for completeness. Note that they are consistent with zero at the 1- and 2-sigma level respectively.

We observe that the data are only roughly consistent with relations (3.14), within two standard deviations. In addition, using relations (3.15), one observes that while the data for $B \rightarrow DK$ seems reasonable, the data for $B \rightarrow D\pi$ prefer a vanishing colour-suppressed amplitude by giving a negative central values for it. While this is on one hand sensible, because this amplitude is expected to be small, it is at odds with the measured non-vanishing CP-asymmetries. Together, these observations lead to a bad χ^2_{min} -value in a global fit to the experimental data, independent of the U -spin breaking parameters.

Observable	Value
$\text{BR}(B^- \rightarrow D^0 \pi^-)$	$(48.4 \pm 1.5) 10^{-4}$
$A_{\text{CP}}(B^- \rightarrow D^0 \pi^-)$	-0.008 ± 0.008
$\frac{\text{BR}(B^- \rightarrow D^0 K^-)}{\text{BR}(B^- \rightarrow D^0 \pi^-)} (*)$	0.076 ± 0.006
$A_{\text{CP}}(B^- \rightarrow D^0 K^-)$	0.07 ± 0.04
$\frac{2\text{BR}(B^- \rightarrow D^0_+ K^-)}{\text{BR}(B^- \rightarrow D^0 K^-)}$	1.10 ± 0.09
$A_{\text{CP}}(B^- \rightarrow D^0_+ K^-)$	0.24 ± 0.07
$\frac{2\text{BR}(B^- \rightarrow D^0_- K^-)}{\text{BR}(B^- \rightarrow D^0 K^-)}$	1.06 ± 0.10
$A_{\text{CP}}(B^- \rightarrow D^0_- K^-)$	-0.10 ± 0.08
$\frac{\text{BR}(B^- \rightarrow D^0_+ K^-)}{\text{BR}(B^- \rightarrow D^0_+ \pi^-)}$	0.086 ± 0.009
$A_{\text{CP}}(B^- \rightarrow D^0_+ \pi^-)$	0.035 ± 0.024
$\frac{\text{BR}(B^- \rightarrow D^0_- K^-)}{\text{BR}(B^- \rightarrow D^0_- \pi^-)}$	0.097 ± 0.017
$A_{\text{CP}}(B^- \rightarrow D^0_- \pi^-)$	0.017 ± 0.026

Table 6: Experimental data for $B^- \rightarrow DK^- / \pi^-$ decays. Data for $B \rightarrow D_{\pm} K$ is taken from the HFAG [19], the rest from the PDG [16]. (*): Error rescaled by the PDG, due to inconsistent measurements.

Checking now in a next step for U -spin breaking by evaluating relations (3.16), we find good agreement in case of the data for $B \rightarrow D^- (K \text{ or } \pi)$, while the relation for the D^0_+ data shows significant U -spin violation, because both CP asymmetries are significantly different from zero and have the same sign. Therefore we expect non-vanishing U -spin breaking parameters in the corresponding fit.

It is interesting to note that for the colour allowed tree decays one may check again naive factorization. In this case the U -spin breaking is given by the ratio of the decay constants, i.e.

$$\frac{\langle B^- | \mathcal{H}_{eff} | D^0 K^- \rangle}{\langle B^- | \mathcal{H}_{eff} | D^0 \pi^- \rangle} = \frac{\lambda}{1 - \frac{\lambda^2}{2}} \left(\frac{1 + y_{1,\epsilon} e^{i\theta_1}}{1 - y_{1,\epsilon} e^{i\theta_1}} \right) \simeq \frac{\lambda}{1 - \frac{\lambda^2}{2}} \frac{f_K}{f_\pi} \sim 0.28. \quad (3.20)$$

In this approach we obtain $\theta_1 = 0$ and $y_{1,\epsilon} \sim 0.1$ from the ratio of the decay constants. The comparison with experiment (see table 6),

$$\sqrt{\frac{\text{BR}(B^- \rightarrow D^0 K^-)}{\text{BR}(B^- \rightarrow D^0 \pi^-)}} = 0.276 \pm 0.011, \quad (3.21)$$

shows excellent agreement, indicating the well known fact that naive factorization works reasonably well in colour-allowed tree decays.

One may use this observation to fix $\theta_1 \equiv 0$, thereby breaking the parametric invariances described above. This results in $\chi^2/\text{d.o.f.} = 7.42/4$, but mostly independent of the assumption concerning U -spin breaking, corresponding to the above discussion. While the results are therefore to be handled with care, we note that the fit prefers large values U -spin breaking parameter r_2 , and leads to the predictions $\text{BR}(B^- \rightarrow \bar{D}^0 K^-) \in [0.02, 0.09] \times 10^{-4}$ and $\text{BR}(B^- \rightarrow \bar{D}^0 \pi^-) \leq 0.03 \times 10^{-4}$ at 1σ .

4 Conclusions

Since methods based on factorization do not seem to converge quickly to allow for a reliable prediction for non-leptonic decays, the method of flavour symmetries looks more promising. Clearly the latter will allow us to perform precision calculations only if we get a reasonable control over symmetry breaking.

Using the full $SU(3)$ flavour symmetry becomes quite complicated once its complete breaking is taken into account. However, the isospin subgroup of full $SU(3)$ may be assumed to be a reasonably good symmetry and hence only the breaking along the “orthogonal” directions in $SU(3)$ space has to be considered.

We have studied the U -spin subgroup of $SU(3)$, which has the advantage that the charge operator commutes with the symmetry generators and hence also the weak hamiltonian for B decays has a simple structure under this symmetry. The breaking term is due to the mass difference between the down and the strange quark and has a simple structure inferred from QCD.

Based on this we have discussed how U -spin breaking can be incorporated on a purely group theoretical basis. We have shown a few applications, in which the U -spin breaking turns out roughly of the order implied by the difference in the decay constants f_π and f_K .

However, the full strength of this strategy can be exploited only in the future. Since the B_d and the B_s form a U -spin doublet, the approach requires information on decay modes which will be gathered in the near future at the LHC. With a sufficient amount of data there will be a chance to obtain control over flavour $SU(3)$ breaking and hence a possible road to precise predictions for non-leptonic decays may be opened.

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